Page 4.

$$
\begin{aligned}
x' &= \frac{ax+b}{cx+d} \\
x_1' - x_2' &= \frac{ax_1+b}{cx_1+d} - \frac{ax_2+b}{cx_2+d} = \frac{(ax_1+b)(cx_2+d)-(cx_1+d)(ax_2+b)}{(cx_1+d)(cx_2+d)} \\
&= \frac{acx_1x_2+adx_1+bcx_2+ad-acx_1x_2-bcx_1-adx_2-bd}{(cx_1+d)(cx_2+d)} \\
&= \frac{(ad-bc)(x_1-x_2)}{(cx_1+d)(cx_2+d)} \\
\frac{x_1'-x_2'}{x_3'-x_2'} &= \frac{x_1-x_2}{x_3-x_2}\frac{(cx_3+d)(cx_2+d)}{(cx_1+d)(cx_2+d)} = \frac{x_1-x_2}{x_3-x_3}\frac{cx_3+d}{cx_1+d} \\
\frac{(x_1'-x_2')/(x_3'-x_2')}{(x_1'-x_4')/(x_3'-x_4')} &= \frac{\frac{x_1-x_2}{x_3-x_2}\cdot\frac{cx_2+d}{cx_1+d}}{\frac{x_1-x_4}{x_3-x_4}\cdot\frac{cx_3+d}{cx_1+d}} = \frac{(x_1-x_2)/(x_3-x_2)}{(x_1-x_4)/(x_3-x_4)}
\end{aligned}
$$

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$$\begin{aligned}
ba &= e \\
bab &= eb = b \\
b(ab) &= b \\
cb &= e \\
(cb)(ab) &= eb \\
e(ab) &= e \\
ab &= e \\
bab &= eb \\
be &= eb
\end{aligned}$$

If $be = db$, then

$$bea = dba \quad \to \quad ba = de \quad \to \quad ba = ed \quad \to \quad ba = d \quad \to \quad e = d.$$

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By definition, $a^2 \in G = e, a, b, c$, so $a^2 = 3$, or $a^2 = b$, or $a^2 = c$. $a^2 = a$ implies $a = e$, contrary to assumption.

$a^2 = b$ will yield a group isomorphic to the group with $a^2 = c$; hence, they have the same group structure. Only two different group structures are left. With two different structures given in the book, they must be the only two possible.

Since the identity element commutes with everything $ea = ae$, then only such products as ab need be considered. $ab = b$ or $ab = a$ implies one is the identity, so by assumption $ab = c$. But $ab \neq ba$ which implies $ba = b$ or $ba = a$. Then, either a or b is the identity, contrary to assumption. Therefore, $ab = ba$ for all a, b and the group is abelian.

Group (A): Rotation of a square about its center, remaining within the plane. $a = \mathrm{Rot}(\pi/2)$.

$$\mathrm{Group(A)} = \{a = 1, b = -1, c = -i, e = 1\}.$$

Group (B): Rotation of a square about a symmetry axis, rotation out of the plane.

$$\mathrm{Group(B)} = \left\{ a = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, b = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, c = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$